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AN ALL DIGITAL LOW DATA RATE COMMUNICATION SYSTEM

by

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Abstract

The advent of digital hardwares has made it feasible to implement many communication system components digitally. With the exception of frequency down conversion, the proposed low data rate communication system uses digital hardwares completely. Although the system is designed primarily for deep space communications with large frequency uncertainty and low signal-to-noise ratio, it is also suitable for other low data rate applications with time-shared operation among a number of channels. Emphasis is placed on the fast Fourier transform receiver and the automatic frequency control via digital filtering. The speed available from the digital system allows sophisticated signal processing to reduce frequency uncertianty and to increase the signal-to-noise ratio. The practical limitations of the system such as the finite register length are examined. It is concluded that the proposed all-digital system is not only technically feasible but also has potential cost reduction over the existing receiving systems.

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I. Introduction

The advent of large scale integration has suggested new approaches to signal processing problems. In the design of communication receivers, the trend has been toward the increased use of digital circuitries. The speed of the present digital hardware, however, has prevented the data rate from being in the range of gegabits per second. For low data rate communication such as between 1 kilobits per second to 1 bit per second or lower, the digital system is not only technically feasible, but also has potential cost reduction over the existing receiving systems. The low data rate communication system we consider here is designed primarily for deep space communications. It is also suitable for other applications as long as the data rate is reasonably low. The digital filters employed in the system can operate fast enough so that it may be time-shared among a number of channels.

For the deep space applications, such as the communication between the earth and a probe in the atmosphere of Venus, the small transmitter power of the probe has constrained the data rate to be low. The large frequency uncertainty caused primarily by the oscillator instability and the doppler effects requires a large IF filter bandwidth and thus the received signal-to-noise ratio is very small. To have an efficient communication, it is necessary to remove such frequency uncertainty. A small amount of frequency error which cannot be removed may be tracked by an automatic frequency control (AFC) loop. In the proposed system, all components are implemented digitally except the frequency down conversion. To perform the mixing operation at 2.3 GHz, the typical operating frequency, by using digital circuitry is not feasible at the present time. Correlation operation in the kilo-hertz frequency range, however, can easily be performed digitally.

In addition to the spectral analysis receiver operation and the frequency and time synchronizations, other problems that have been considered are the multipath fading effects [1], [2], coding and modulation [3], [4]. A general discussion of the low data rate communication system design is given by Chadwick and Springett [5]. The fast Fourier transform (FFT) receiver performance was considered by Goldstein [6] and Ferguson [7]. More recently, Holmes [8] examined the optimum nonchoerent receiver at low data rate for unknown doppler shifted signals. The unrealizable filters required in the receiver can be implemented by using digital filters.

There are practical limitations on the digital systems. For example, the computer word length is finite and the A/D conversion error and the round off error are unavoidable. Such limitations will be examined in detail.

II. System Configuration and FFT Receiver

The proposed system is shown in Fig. 1. With the exception of the frequency down conversion and the local oscillator, the receiving system can be implemented digitally. The input to the A/D converter can have a frequency of several hundred hertz to a few thousand hertz depending on signalling frequency and the uncompensated frequency uncertainty due to oscillator instability and doppler frequency variations. The sampling rate should be at least twice of the largest frequency of the input signal. Experience has indicated that the desirable sampling rate be four or five times of the signal frequency. The digitized data may be recorded in magnetic tape to guard against the loss of data due to system failure such as loss of lock in time or frequency synchronization. Digital filtering of the digitized data reduces the noise in the received signal. This operation is optional if the signal-to-noise ratio is high but is essential if the signal-to-noise ratio is low. The discrete Fourier transform (DFT) of the data is performed by using the fast Fourier transform (FFT). The square of the magnitude of DFT is proportional to the power spectrum.

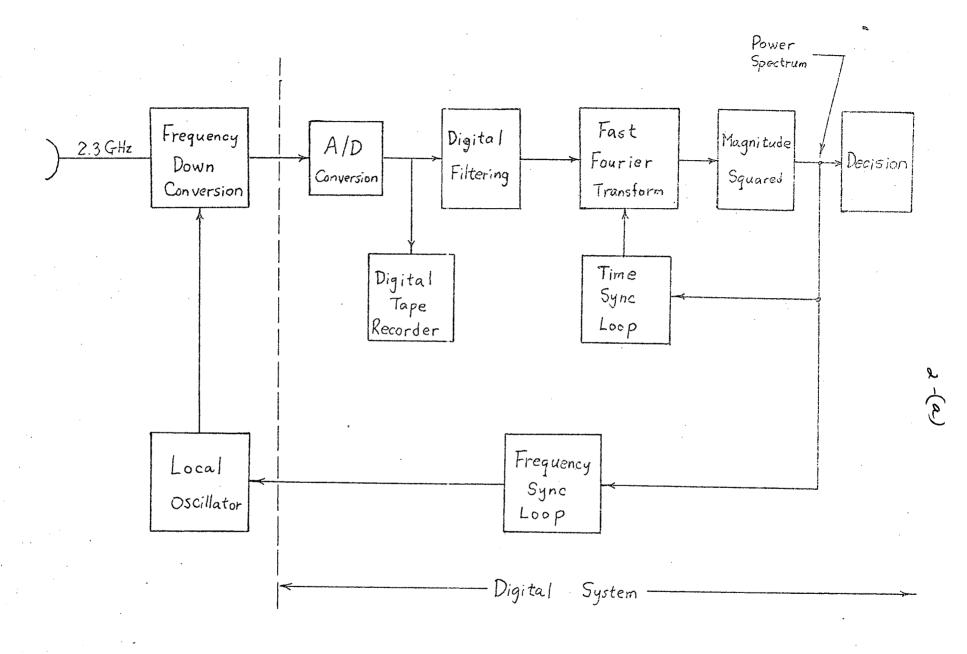


Fig. 1 Proposed Low Data Rate Communication System

The decision is based on the frequency of the largest spectral component. In the absence of noise, the receiver will always select the correct frequency. However, the magnitude of the measured spectral peak depends on the word timing error. In the presence of noise, time sync. loop provides the work tracking. The frequency sync. loop determines the frequency drift and provides an up-to-date estimate of the actual frequency. The local oscillator frequency can then be adjusted according to the frequency estimate. As the data rate is low, there is sufficient time for on-line, i.e real-time, operation of the complete system to provide continuous frequency and time tracking and the signal-to-noise ratio improvement via digital filtering.

The fast Fourier spectral analysis can be very sensitive to the signal-to-noise ratio. Consider two FSK signals of frequencies 100 Hz and 200 Hz. Each bit of the received data contains one of the two signals plus an additive Gaussian noise. The bit duration is 1 sec. and the sampling rate is 1000 samples per second. For signal-to-noise ratios $(S/N)_{\hat{i}} = 0.1$ and 0.01, the amplitude spectrum of the 100 Hz signal is shown in Fig. 2. The signal can be detected correctly from the spectral peak at $(S/N)_{\hat{i}} = 0.1$. As $(S/N)_{\hat{i}}$ is decreased to 0.01 the signal cannot be detected correctly. A possible interpretation of this result is that there may be some threshold signal-to-noise ratio below which the signal is no longer detectable. However an analysis of the noncoherent FSK indicates that such threshold does not exist and the error rate is approximately 0.48 at $(S/N)_{\hat{i}} = 0.01$. For better error rate, it is necessary to do some signal processing to increase the signal-to-noise ratio. Let N be the number of samples. By doubling the sampling rate we can use twice as many signal samples per spectrum. The adjacent points in the power spectrum can be averaged to give [6],

$$P(n) = \left| \frac{1}{N} \sum_{i=0}^{2N-1} x_i \exp(-j \frac{\pi}{N} in) \right|^2 + \frac{1}{N} \sum_{i=1}^{2N-1} x_i \exp[-j \frac{\pi}{N} i(n+1)] \right|^2, \quad (1)$$

$$n = 0, 2, 4, ..., N$$



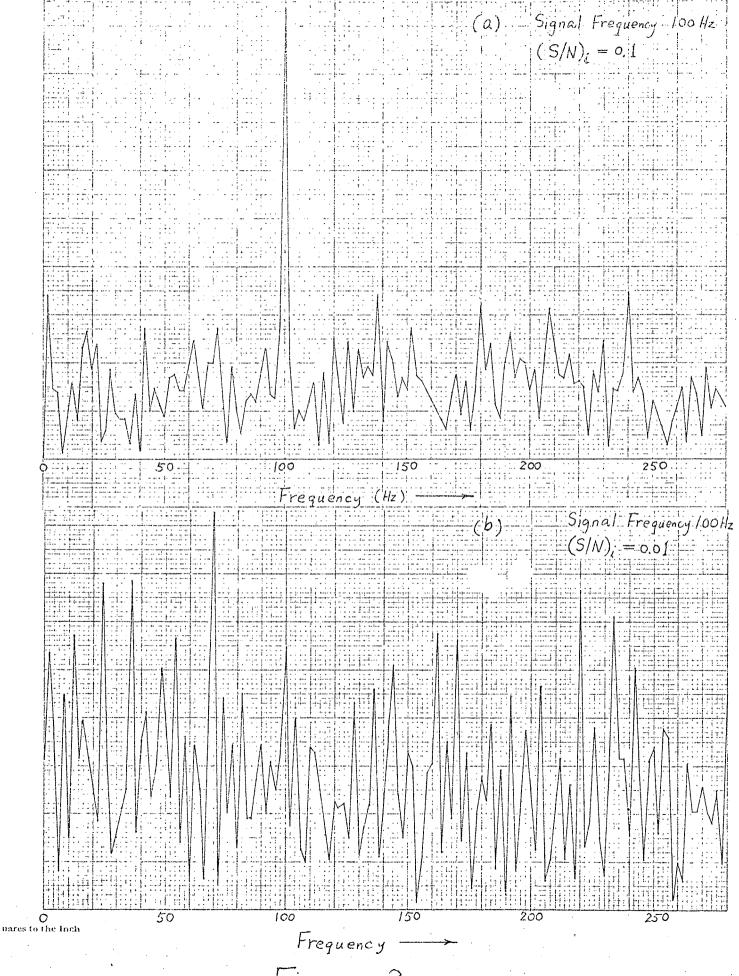


Figure 2

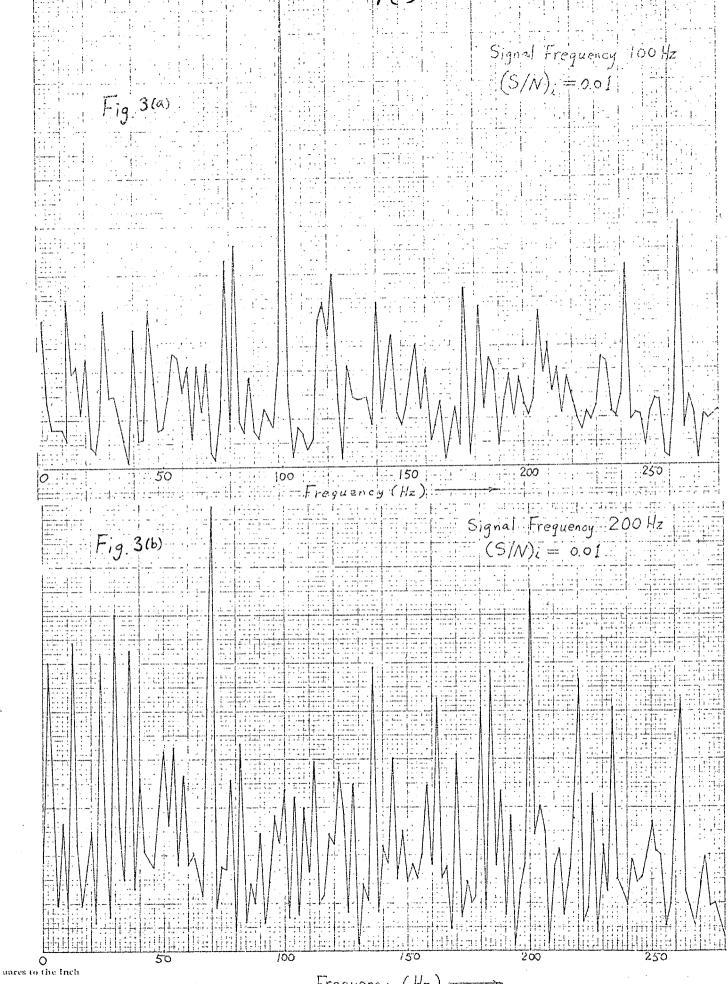
By using Eq. (1), the spectral peak at 100 Hz can be detected correctly as shown in Fig. 3(a). If the signal frequency is 200 Hz, it cannot be determined correctly (Fig. 3(b)) but there is a strong frequency component at 200 Hz. The FFT receiver usually has more difficulty to detect the highest signal frequency component unless the sampling rate is much higher than such frequency. In spite of some practical problems with the FFT receiver as described above, the use of sophisticated signal processing at low data rate makes the FFT receiver a very feasible system for deep space communications. Futhermore, the FFT hardware presently available has a speed several times faster than the use of FFT software. Such improvement in processing time from using digital devices is particularly important for real-time applications.

The probability of error of the fast Fourier transform receiver is the same as the optimum noncoherent MFSK (multiple frequency shift keyed). If sampling, quantization and finite word length effects are considered, the performance will no longer be optimum. The sampling rate can always be chosen to be large enough to have negligible effect. The problems of quantization and finite word length will be discussed later.

III. Digital Filter Implementation of Automatic Frequency Control (AFC) Loop

A large amount of frequency drifts that cause uncertainty may be removed by prior knowledge and prediction. However, other drifts will surely remain, and it is necessary to track these in any practical system. Several techniques of frequency tracking have been proposed. Goldstein's technique [6] obtains a frequency discriminator characteristic (S curve) by taking the difference, F_n , of the two terms in Eq. (1) as an estimate of the current frequency error,

$$F_n = r_{n+1} - r_n \tag{2}$$



Frequency (Hz)

where

$$r_n = \left| \frac{1}{N} \sum_{i=0}^{2N-1} x_i \exp(-j \frac{\pi}{N} in) \right|^2$$

is the nth spectral line. Let T be the signal duration. The nominal signal frequency here is $(n+\frac{1}{2})/T$, centered between two adjacent spectral lines. F_n is filtered and used to correct the local oscillator tuning as in any frequency-locked loop. The procedure is simple as it involves only FFT. One practical problem, however, is that F_n is not necessarily equal to zero even if there is no frequency drift and no noise. Furthermore the method is suitable for very small frequency drift which may not be the case in practice.

The second technique due to Ferguson [7] is to use a weighted average of the k closest spectral components, where k is some small integer. If the received signal is norminally at frequency n/T, then the estimate is

$$F_n = C \sum_{i=-k/2}^{k/2} a_i r_{n+i}$$
 (3)

where C is a normalizing constant and the a 's are a set of linearizing weighting coefficients. This technique of course is highly dependent on the choice of the a 's.

The third technique due to Chadwick [5] is based on the spectral lines nearest to the observed frequency. Let r_0 be the spectral component of the observed frequency and r_{+1} and r_{-1} be the adjacent spectral lines with frequency 1/T Hz larger and smaller, respectively, than the observed frequency. The frequency estimate is

$$F_{n} = \frac{r_{+1} - r_{-1}}{2r_{0}T}.$$
 (4)

which tends to have less accuracy than Eq. (3) and is useful only to very small frequency drifts.

The fourth technique implements the automatic frequency control (AFC) loop by using digital filters. The resulting digital loop is incorporated in the system

shown in Fig. 1. The method as originally proposed by Ferguson [9] is essentially a discrete frequency-locked loop. A general block diagram of the AFC network is shown in Fig. 4. The digital filter disciminator measures the frequency error with the output proportional to the frequency drift, which is the difference between the incoming signal frequency $\omega(\ell)$ at the ℓ th sampling instant and the filtered output frequency $\hat{\omega}(\ell)$. Fig. 5 is the block diagram of the digital filter discriminator. Here the difference of the squares of the two filter outputs is averaged over kT seconds where T is the sampling interval and k is sufficiently large. The output of the discriminator is then the smoothed frequency drift and the noise can further be suppressed by loop filtering. The pulse transfer functions of the two digital filters shown in Fig. 5 are

$$H_{1}(z) = \frac{Y_{1}(z)}{X(z)} = \frac{G(1 - 2z^{-2} + z^{-1})}{(1 - a_{1}z^{-1} - b_{1}z^{-2})(1 - a_{2}z^{-1} - b_{2}z^{-2})}$$

$$H_{2}(z) = \frac{Y_{2}(z)}{X(z)} = \frac{G(1 - 2z^{-2} + z^{-1})}{(1 - a_{3}z^{-1} - b_{3}z^{-2})(1 - a_{4}z^{-1} - b_{4}z^{-2})}$$
(5)

where the coefficients a_i , b_i and G are tabulated in Fig. 5. The discriminator transfer characteristic as shown in Fig. 6 is a plot of $|H_2(e^{j\omega T})|^2 - |H_1(e^{j\omega T})|^2$ versus frequency. Such characteristic is very similar to that of the Foster-Seely discriminator for FM demodulation. An important advantage of the digital discriminator (Fig. 5) is that it can easily be constructed with digital hardware. Its performance is independent of the accuracy of the fast Fourier transform, which the three techniques discussed above completely depend on. The bandwidths of the two filters can be increased to allow larger frequency drift as long as the linearity of the discriminator characteristic can be maintained. This is another advantage over the other techniques which use FFT for frequency tracking.

Consider Fig. 4 again. Let the loop gain and the discriminator slope be

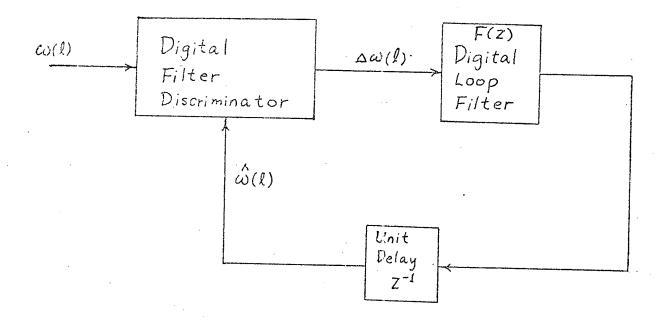


Fig. 4 General Block Diagram of Digital Filter AFC

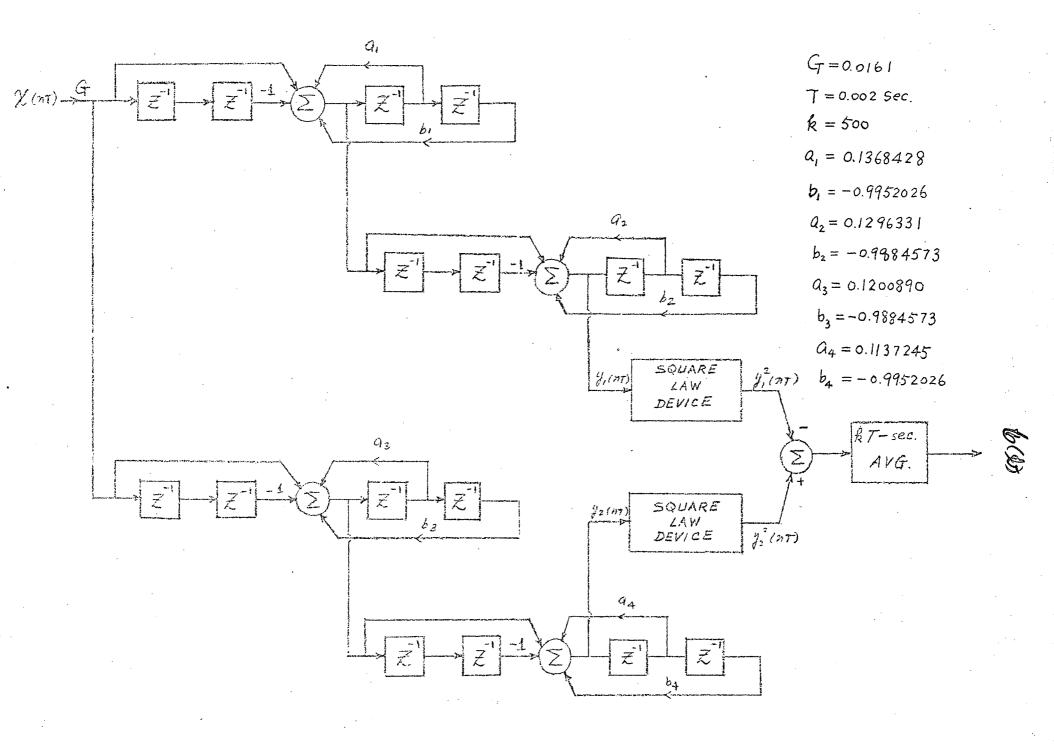


Fig. 5 BLOCK DIAGRAM OF DIGITAL FILTER DISCRIMINATOR

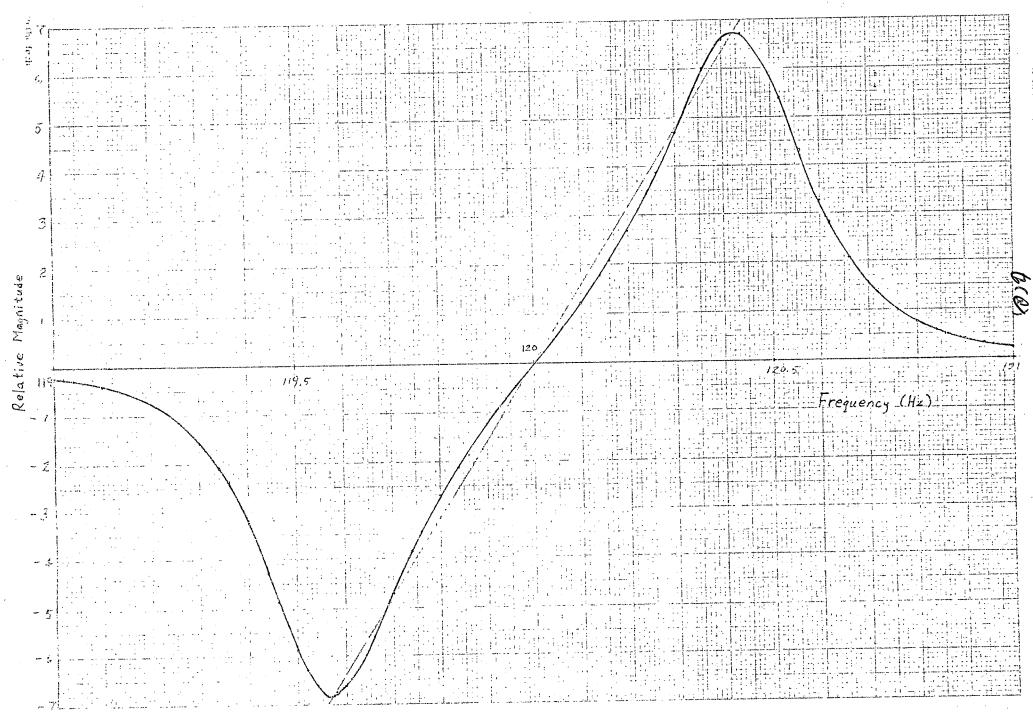


Fig. 6 Digital Discriminator Transfer Characteristic

combined into a single loop gain K. Also let

$$F(z) = \frac{1}{(1 - bz^{-1})(1 - z^{-1})}$$
 (6)

and σ^2 be the variance of the additive noise at the discriminator input. Then the variance of the noisy estimate of the frequency error, $\Delta\omega(\ell)$ is derived as

$$\sigma_{\Delta}^{2} = \frac{2K^{2}\sigma^{2}}{(1-b)[2(1+b)-K]} \tag{7}$$

which, with typical loop parameters can be shown to be smaller than the variance of F_n given by Eq. (4).

One problem² that may arise with the digital filter discriminator is the transient phenomenon during frequency sweeping. One solution to this problem is to operate the loop at the slightly underdamped condition even though the loop may then have larger probability of losing lock. It is noted, however, that the transient period for the loop to adjust itself to the new frequency may be kept small by proper loop design.

The time sync loop can be implemented digitally with a similar method described above. Digital hardware for word tracking is commercially available.

IV. Effects of Quantization and Finite Word Length

Consider first the A/D (analog to digital) conversion error. Let \mathbb{E}_{0} be the quantization level. The mean-square error, i.e. the noise variance, caused by the A/D conversion for the AFC loop is

$$\sigma_{\mathbf{f}}^{2} = \frac{E_{0}^{2}}{12} \frac{1+b}{1-b} \frac{K}{[2(1+b)-K]}$$
 (8)

For typical loop parameters b = 0.725 and K = 0.0241, $\sigma_{\rm f}^2$ = (0.044)E $_{\rm o}^2$ /12, which is quite small. Let us assume that for the smallest input signal that is expected to pass through the digital filter the filter output must be 40 dB higher than the noise level created by quantization. We assume further that the digital filter

 $^{^{1}}$ Eq. (4) of Ref. 9 is incorrect.

Private communication with M.J. Ferguson, February 1973.

must process data over a 40 dB range of input amplitudes. The output register must then have a range of at least $(0.044/12)^{1/2}(100)(100)E_0 = 605 E_0$. The output-register word length required is therefore 10 bits, which can easily be met in practice.

In the above discussion we have assumed that the digital filter discriminator characterized by Eq. (5) is noiseless. Although this assumption is not valid rigorously speaking, exact computation of both A/D and roundoff noise variances in the discriminator can be very tedious. Such variances, however, only cause some inaccuracy in the parameter K. Let the noise variance of K be σ_K^2 . σ_K^2 is equal to the sum of the A/D and roundoff noise variances in the discriminator. The noise variance at the AFC loop output resulting from σ_K^2 is

$$\sigma_{r}^{2} = \frac{2K^{2}\sigma_{K}^{2}}{(1-b)[2(1+b)-K]} = 0.01115 \sigma_{K}^{2}$$
(9)

The total A/D and roundoff noise variance is equal to the sum of $\sigma_{\rm r}^{\ 2}$, $\sigma_{\rm f}^{\ 2}$ and the roundoff error due to the parameter b. The poles of the narrowband digital filters given by Eq. (5) are located very closely to the unit circle. The A/D and roundoff errors for both filters can be very large. The structure of the filters shown in Fig. 5, however, provides the minimum roundoff errors. A worst estimate of the discriminator noise variance may take the value $\sigma_{\rm K}^{\ 2} \leq 100~{\rm E}_{\rm O}^{\ 2}$. Then with 40 dB signal-to-quantization noise ratio and the 40 dB dynamic range as mentioned above, a commercially available 14 bit A/D converter is quite adequate for the applications described in this paper.

In addition to the roundoff error, the finite register length also causes the parameter quantization such that the filter coefficients cannot be specified exactly. Kaiser [10] has shown that for a filter with clustered poles a series or parallel combination of first-and second-order sections provides more accuracy in the pole positions than a direct form realization. Thus the filter configuration shown in Fig. 5 is least sensitive to the parameter quantization.

Now we consider the roundoff noise effect in FFT operation. For N-point FFT, the output noise variance is given by [11]

$$\sigma_{E}^{2} \simeq 4N(\frac{E_{0}^{2}}{12}) \tag{10}$$

which is proportional to N. The variance can be reduced by scaling, such as the multiplication factor of 1/2, at each stage of FFT. Scaling, of course, requires a slight increase in the FFT hardware. With scaling, the variance becomes

$$\sigma_{\rm E}^2 = 20(\frac{{\rm E}_{\rm o}^2}{12})$$
 (11)

but the noise-to-signal ratio is 5NE_0^2 which is still proportional to N. In low data rate communication, a typical value for N is 1024. To achieve a signal-to-noise ratio of 40 dB, the required register length is 13 bits. If the signal dynamic range is also considered, much longer register length is needed. Thus the number of points that can effectively be performed by a given FFT hardware or software is limited. This may also provide an interpretation of the "threshold" behavior of the signal detectability in FFT receiver.

V. Concluding Remarks

With the present technology in digital devices, we have shown that the proposed all-digital low data rate communication system is completely feasible from both performance and economy viewpoints. The effects of the quantization and finite word length on the digital system performance are normally very significant. It is shown, in this paper, however, that by properly designing the digital filters such effects can be minimized. A commercially available 14 bit register length is adequate to provide a signal-to-noise ratio of over 40 dB in both digital filter and FFT. With continued improvement in the digital hardwares, there is every reason to believe that the proposed system provides cost-reduction over existing receiver systems.

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